(* Calculation rule in Mathematica-syntax for the equioscillation points z_j= y_j from [-1,1]\{-1,1} at which a proper (monic)
Zolotarev polynomial $Z_{n,s}(x)$ attains the values $-||Z_{n,s}||_\infty$;
consider for n odd the preliminary tentative form as given in Formula (11)
of [6]. The case of n even and the case of equioscillation points z_j= x_j (see [48, (36),(37)] in [6]) can be treated similarly;
compare also with Section 2.2 in in the present ZFP Repository [55] of [6] *)

In[•]:= (* The said Formula (11) reads for n=5 respectively n=7 as follows *)

In[•]:= $$S_{5,\alpha,\beta,y_j}[x] = \left(-1+x^2\right)(x-y1)^2(x-\alpha)-\frac{1}{2}(-y1+\beta)^2(-\alpha+\beta)\left(-1+\beta^2\right)$$

respectively

In[•]:= $$S_{7,\alpha,\beta,y_j}[x] = \left(-1+x^2\right)(x-y1)^2(x-y2)^2(x-\alpha)-\frac{1}{2}(-y1+\beta)^2(-y2+\beta)^2(-\alpha+\beta)\left(-1+\beta^2\right)$$

(* Our Mathematica-based calculation rule for the y_j
(compare with Theorem 1 (ii) of [48] in [6]) reads as follows *)

```
so[α_, β_, k_] = 1 / 2 (α^k + β^k - 1 - (-1) ^k);
sso[α_, β_, k_] = 1 / 2 (α^k + β^k + 1 + (-1) ^k);
Fo[α_, β_, 0] = 1;
Fo[α_, β_, k_?Negative] = 0;
                ⌊negativ
Fo[α_, β_, k_] := (-1) ^k / k!
   Det[Table[If[j > i, If[j == i + 1, i, 0], so[α, β, i - j + 1]], {i, k}, {j, k}]];
  ⌊De·· ⌊Tabelle ⌊wenn   ⌊wenn
FFo[α_, β_, k_?Negative] = 0;
                 ⌊negativ
FFo[α_, β_, 0] = 1;
FFo[α_, β_, k_] := (-1) ^k / k!
   Det[Table[If[j > i, If[j == i + 1, i, 0], sso[α, β, i - j + 1]], {i, k}, {j, k}]];
  ⌊De·· ⌊Tabelle ⌊wenn   ⌊wenn
ψ2[n_] :=
  Module[{m = (n - 1) / 2}, Det[Table[FFo[α, β, m + i - j + 1], {i, m - 1}, {j, m - 1}]]];
 ⌊Modul                     ⌊De·· ⌊Tabelle
ψ2b[n_, k_] := Module[{m = (n - 1) / 2}, Det[Table[
               ⌊Modul                    ⌊De·· ⌊Tabelle
    If[j == k, -FFo[α, β, m + i - j + k + 1], FFo[α, β, m + i - j + 1]], {i, m - 1}, {j, m - 1}]]]
   ⌊wenn
```

(* Our Mathematica-
based calculation rule for the x_j (compare with Theorem 1 (iii) of [48] in [6])
would require (for n odd) the following alternative two last command lines *)

```
ψ3[n_] := Module[{m = (n - 1) / 2}, Det[Table[Fo[α, β, m + i - j], {i, m}, {j, m}]]];
         ⌊Modul                    ⌊De·· ⌊Tabelle
ψ3b[n_, k_] := Module[{m = (n - 1) / 2},
               ⌊Modul
  Det[Table[If[j == k, -Fo[α, β, m + i - j + k], Fo[α, β, m + i - j]], {i, m}, {j, m}]]]
 ⌊De·· ⌊Tabelle ⌊wenn
```

(* Thus we have at hand a Mathematica-
based calculation rule to determine all equioscillations points
(in the interior of [-1,1]) of a proper monic Zolotarev polynomial of odd degree *)

(* Executing the calculation rule for n=
n_0=5 amounts here to (note that for n=5 the polynomial
in Theorem 1 (ii) of [48] in [6] is linear in y) *)

In[∘]:= y * ψ2[5] + ψ2b[5, 1]

Out[∘]=

$$\frac{1}{6} y \left(\frac{3\alpha}{2} - \frac{3\alpha^3}{8} + \frac{3\beta}{2} + \frac{3\alpha^2\beta}{8} + \frac{3\alpha\beta^2}{8} - \frac{3\beta^3}{8}\right) + \frac{1}{24}\left(3 - \frac{3\alpha^2}{2} + \frac{15\alpha^4}{16} + 3\alpha\beta - \frac{3\alpha^3\beta}{4} - \frac{3\beta^2}{2} - \frac{3\alpha^2\beta^2}{8} - \frac{3\alpha\beta^3}{4} + \frac{15\beta^4}{16}\right)$$

(* Solving for y yields) *)

In[∘]:= Solve[y * ψ2[5] + ψ2b[5, 1] == 0, y]
löse

Out[∘]=

$$\left\{\left\{y \to \frac{16 - 8\alpha^2 + 5\alpha^4 + 16\alpha\beta - 4\alpha^3\beta - 8\beta^2 - 2\alpha^2\beta^2 - 4\alpha\beta^3 + 5\beta^4}{8\left(-4\alpha + \alpha^3 - 4\beta - \alpha^2\beta - \alpha\beta^2 + \beta^3\right)}\right\}\right\}$$

(* Inserting this y for y1 in S_{5,α,β,y_j}[x] yields a quintic polynomial in x *)

In[∘]:=

$$\left(\left(-1 + x^2\right)(x - y1)^2 (x - \alpha) - \frac{1}{2}(-y1 + \beta)^2 (-\alpha + \beta)\left(-1 + \beta^2\right)\right) /. \; y1 \to \frac{16 - 8\alpha^2 + 5\alpha^4 + 16\alpha\beta - 4\alpha^3\beta - 8\beta^2 - 2\alpha^2\beta^2 - 4\alpha\beta^3 + 5\beta^4}{8\left(-4\alpha + \alpha^3 - 4\beta - \alpha^2\beta - \alpha\beta^2 + \beta^3\right)} \; // \text{Simplify}$$

vereinfache

Out[∘]=

$$\frac{(\alpha - \beta)\left(-1 + \beta^2\right)\left(16 + 5\alpha^4 - 12\alpha^3\beta + 24\beta^2 - 3\beta^4 + 4\alpha\beta\left(12 + \beta^2\right) + \alpha^2\left(-8 + 6\beta^2\right)\right)^2}{128\left(\alpha^3 - \alpha^2\beta + \beta\left(-4 + \beta^2\right) - \alpha\left(4 + \beta^2\right)\right)^2} + \left(-1 + x^2\right)(x - \alpha)\left(x + \frac{-16 - 5\alpha^4 + 4\alpha^3\beta + 8\beta^2 - 5\beta^4 + 4\alpha\beta\left(-4 + \beta^2\right) + 2\alpha^2\left(4 + \beta^2\right)}{8\left(\alpha^3 - \alpha^2\beta + \beta\left(-4 + \beta^2\right) - \alpha\left(4 + \beta^2\right)\right)}\right)^2$$

(* The list of the coefficients of this quintic polynomial is *)

In[*]:=
```
CoefficientList[%, x] // Simplify
```
Liste der Koeffizienten   vereinfache

Out[*]=

$$\Big\{\Big(\big(2\alpha\,(-16-5\alpha^4+4\alpha^3\beta+8\beta^2-5\beta^4+4\alpha\beta(-4+\beta^2)+2\alpha^2(4+\beta^2))\big)^2+(\alpha-\beta)(-1+\beta^2)\big(16+5\alpha^4-12\alpha^3\beta+24\beta^2-3\beta^4+4\alpha\beta(12+\beta^2)+\alpha^2(-8+6\beta^2)\big)^2\Big)\Big/\Big(128\big(\alpha^3-\alpha^2\beta+\beta(-4+\beta^2)-\alpha(4+\beta^2)\big)^2\Big),$$

$$-\Big(\Big(\big(16+21\alpha^4-20\alpha^3\beta-8\beta^2+5\beta^4+12\alpha\beta(-4+\beta^2)-18\alpha^2(4+\beta^2)\big)\big(16+5\alpha^4-4\alpha^3\beta-8\beta^2+5\beta^4-4\alpha\beta(-4+\beta^2)-2\alpha^2(4+\beta^2)\big)\Big)\Big/\Big(64\big(\alpha^3-\alpha^2\beta+\beta(-4+\beta^2)-\alpha(4+\beta^2)\big)^2\Big)\Big),$$

$$\alpha-\frac{-16-5\alpha^4+4\alpha^3\beta+8\beta^2-5\beta^4+4\alpha\beta(-4+\beta^2)+2\alpha^2(4+\beta^2)}{4\big(\alpha^3-\alpha^2\beta+\beta(-4+\beta^2)-\alpha(4+\beta^2)\big)}-\frac{\alpha\big(-16-5\alpha^4+4\alpha^3\beta+8\beta^2-5\beta^4+4\alpha\beta(-4+\beta^2)+2\alpha^2(4+\beta^2)\big)^2}{64\big(\alpha^3-\alpha^2\beta+\beta(-4+\beta^2)-\alpha(4+\beta^2)\big)^2},$$

$$-1-\frac{\alpha\big(-16-5\alpha^4+4\alpha^3\beta+8\beta^2-5\beta^4+4\alpha\beta(-4+\beta^2)+2\alpha^2(4+\beta^2)\big)}{4\big(\alpha^3-\alpha^2\beta+\beta(-4+\beta^2)-\alpha(4+\beta^2)\big)}+\frac{\big(-16-5\alpha^4+4\alpha^3\beta+8\beta^2-5\beta^4+4\alpha\beta(-4+\beta^2)+2\alpha^2(4+\beta^2)\big)^2}{64\big(\alpha^3-\alpha^2\beta+\beta(-4+\beta^2)-\alpha(4+\beta^2)\big)^2},$$

$$\frac{-16-9\alpha^4+8\alpha^3\beta+8\beta^2-5\beta^4+6\alpha^2(4+\beta^2)}{4\big(\alpha^3-\alpha^2\beta+\beta(-4+\beta^2)-\alpha(4+\beta^2)\big)},\ 1\Big\}$$

(* Here $\frac{-16-9\alpha^4+8\alpha^3\beta+8\beta^2-5\beta^4+6\alpha^2(4+\beta^2)}{4(\alpha^3-\alpha^2\beta+\beta(-4+\beta^2)-\alpha(4+\beta^2))}=-5s$ **must hold,**
hier

hence $s=\frac{1}{20}\left(5\alpha-\frac{8(1+\alpha)}{2+\alpha-\beta}+5\beta+\frac{8-8\alpha}{2-\alpha+\beta}+\frac{4(-1+\alpha^2)}{\alpha+\beta}\right)$ ***)**

(* In this way one obtains the tentative proper monic Zolotarev
eingegebenes Objekt
polynomial of degree 5 in x, $S_{5,\alpha,\beta}$[x], which depends solely on α and β (see the 2nd algorithm in [6],in particular Formula (12))*)

Out[⋅]=

$$-5 s x^4 + x^5 - \left(x \left(16 + 21 \alpha^4 - 20 \alpha^3 \beta - 8 \beta^2 + 5 \beta^4 + 12 \alpha \beta \left(-4 + \beta^2\right) - 18 \alpha^2 \left(4 + \beta^2\right)\right) \left(16 + 5 \alpha^4 - 4 \alpha^3 \beta - 8 \beta^2 + 5 \beta^4 - 4 \alpha \beta \left(-4 + \beta^2\right) - 2 \alpha^2 \left(4 + \beta^2\right)\right)\right) \Big/ \left(64 \left(\alpha^3 - \alpha^2 \beta + \beta \left(-4 + \beta^2\right) - \alpha \left(4 + \beta^2\right)\right)^2\right) +$$
$$x^3 \left(-1 - \frac{\alpha \left(-16 - 5 \alpha^4 + 4 \alpha^3 \beta + 8 \beta^2 - 5 \beta^4 + 4 \alpha \beta \left(-4 + \beta^2\right) + 2 \alpha^2 \left(4 + \beta^2\right)\right)}{4 \left(\alpha^3 - \alpha^2 \beta + \beta \left(-4 + \beta^2\right) - \alpha \left(4 + \beta^2\right)\right)} + \frac{\left(-16 - 5 \alpha^4 + 4 \alpha^3 \beta + 8 \beta^2 - 5 \beta^4 + 4 \alpha \beta \left(-4 + \beta^2\right) + 2 \alpha^2 \left(4 + \beta^2\right)\right)^2}{64 \left(\alpha^3 - \alpha^2 \beta + \beta \left(-4 + \beta^2\right) - \alpha \left(4 + \beta^2\right)\right)^2}\right) +$$
$$x^2 \left(\alpha - \frac{-16 - 5 \alpha^4 + 4 \alpha^3 \beta + 8 \beta^2 - 5 \beta^4 + 4 \alpha \beta \left(-4 + \beta^2\right) + 2 \alpha^2 \left(4 + \beta^2\right)}{4 \left(\alpha^3 - \alpha^2 \beta + \beta \left(-4 + \beta^2\right) - \alpha \left(4 + \beta^2\right)\right)} - \frac{\alpha \left(-16 - 5 \alpha^4 + 4 \alpha^3 \beta + 8 \beta^2 - 5 \beta^4 + 4 \alpha \beta \left(-4 + \beta^2\right) + 2 \alpha^2 \left(4 + \beta^2\right)\right)^2}{64 \left(\alpha^3 - \alpha^2 \beta + \beta \left(-4 + \beta^2\right) - \alpha \left(4 + \beta^2\right)\right)^2}\right) +$$
$$\left(2 \alpha \left(-16 - 5 \alpha^4 + 4 \alpha^3 \beta + 8 \beta^2 - 5 \beta^4 + 4 \alpha \beta \left(-4 + \beta^2\right) + 2 \alpha^2 \left(4 + \beta^2\right)\right)^2 + (\alpha - \beta) \left(-1 + \beta^2\right) \left(16 + 5 \alpha^4 - 12 \alpha^3 \beta + 24 \beta^2 - 3 \beta^4 + 4 \alpha \beta \left(12 + \beta^2\right) + \alpha^2 \left(-8 + 6 \beta^2\right)\right)^2\right) \Big/ \left(128 \left(\alpha^3 - \alpha^2 \beta + \beta \left(-4 + \beta^2\right) - \alpha \left(4 + \beta^2\right)\right)^2\right)$$

(* For Example,
For-Schleife
if $s=s_0=\frac{41}{25\sqrt{145}}$ is assumed one gets from Proposition 2.4 or 2.5 in [6] that $\alpha=\alpha_0=\frac{67}{5\sqrt{145}}$ and $\beta=\beta_0=\frac{77}{5\sqrt{145}}$, see also Appendix 8.1 in [6]; reversely, with those values of α and β one gets back the value of $s=s_0$ *)

$$\frac{1}{20} \left(5 \alpha - \frac{8 (1 + \alpha)}{2 + \alpha - \beta} + 5 \beta + \frac{8 - 8 \alpha}{2 - \alpha + \beta} + \frac{4 \left(-1 + \alpha^2\right)}{\alpha + \beta}\right) /.\ \alpha \to \frac{67}{5 \sqrt{145}} /.\ \beta \to \frac{77}{5 \sqrt{145}} \ // \ \text{Simplify}$$
vereinfache

Out[⋅]=

$$\frac{41}{25 \sqrt{145}}$$

(* The proper monic quintic Zolotarev polynomial S_{5,α_0,β_0}[x]= $S_{5,\frac{67}{5\sqrt{145}},\frac{77}{5\sqrt{145}}}$[x]=$Z_{5,\frac{41}{25\sqrt{145}}}$[x]=$Z_{5,s_0}$ thus reads as follows *)

Out[⋅]=

$$-\frac{418129}{525625 \sqrt{145}} + \frac{1573 x}{3625} + \frac{137302 x^2}{18125 \sqrt{145}} - \frac{5198 x^3}{3625} - \frac{41 x^4}{5 \sqrt{145}} + x^5$$

(* now consider the degree $n=n_0=7$ *)

(* Executing the above calculation rule for n = 7 and solving for y yields
(note that for n=7 the polynomial in Theorem 1 (ii) of [48] is quadratic in y) *)

In[∘]:=
```
Solve[y^2 ψ2[7] + y ψ2b[7, 1] + ψ2b[7, 2] == 0, y]
```
löse

Out[∘]=

$$\Big\{\Big\{y \to$$

$$\Big(\frac{\alpha}{128}-\frac{\alpha^3}{128}-\frac{\alpha^5}{1024}+\frac{7\,\alpha^7}{2048}-\frac{7\,\alpha^9}{32768}+\frac{\beta}{128}+\frac{\alpha^2\,\beta}{128}-\frac{13\,\alpha^4\,\beta}{1024}+\frac{13\,\alpha^6\,\beta}{2048}+\frac{17\,\alpha^8\,\beta}{32768}+\frac{\alpha\,\beta^2}{128}+\frac{7\,\alpha^3\,\beta^2}{512}-$$

$$\frac{17\,\alpha^5\,\beta^2}{2048}+\frac{5\,\alpha^7\,\beta^2}{8192}-\frac{\beta^3}{128}+\frac{7\,\alpha^2\,\beta^3}{512}-\frac{3\,\alpha^4\,\beta^3}{2048}-\frac{23\,\alpha^6\,\beta^3}{8192}-\frac{13\,\alpha\,\beta^4}{1024}-\frac{3\,\alpha^3\,\beta^4}{2048}+\frac{31\,\alpha^5\,\beta^4}{16384}-$$

$$\frac{\beta^5}{1024}-\frac{17\,\alpha^2\,\beta^5}{2048}+\frac{31\,\alpha^4\,\beta^5}{16384}+\frac{13\,\alpha\,\beta^6}{2048}-\frac{23\,\alpha^3\,\beta^6}{8192}+\frac{7\,\beta^7}{2048}+\frac{5\,\alpha^2\,\beta^7}{8192}+\frac{17\,\alpha\,\beta^8}{32768}-\frac{7\,\beta^9}{32768}-$$

$$\frac{1}{32768}\Big(\sqrt{\big(524288-1638400\,\alpha^2+1310720\,\alpha^4+540672\,\alpha^6-995328\,\alpha^8+182784\,\alpha^{10}+}$$

$$71680\,\alpha^{12}+5184\,\alpha^{14}-408\,\alpha^{16}+7\,\alpha^{18}+393216\,\alpha\,\beta-1572864\,\alpha^3\,\beta+1671168\,\alpha^5\,\beta+$$

$$262144\,\alpha^7\,\beta-1076224\,\alpha^9\,\beta+272384\,\alpha^{11}\,\beta+65408\,\alpha^{13}\,\beta-256\,\alpha^{15}\,\beta-42\,\alpha^{17}\,\beta-$$

$$1638400\,\beta^2+4718592\,\alpha^2\,\beta^2-4210688\,\alpha^4\,\beta^2+245760\,\alpha^6\,\beta^2+1884672\,\alpha^8\,\beta^2-$$

$$974848\,\alpha^{10}\,\beta^2+48832\,\alpha^{12}\,\beta^2+10176\,\alpha^{14}\,\beta^2-17\,\alpha^{16}\,\beta^2-1572864\,\alpha\,\beta^3+$$

$$3997696\,\alpha^3\,\beta^3-2097152\,\alpha^5\,\beta^3-1019904\,\alpha^7\,\beta^3+1509376\,\alpha^9\,\beta^3-521472\,\alpha^{11}\,\beta^3-$$

$$16128\,\alpha^{13}\,\beta^3+784\,\alpha^{15}\,\beta^3+1310720\,\beta^4-4210688\,\alpha^2\,\beta^4+5169152\,\alpha^4\,\beta^4-232448\,\alpha^6$$

$$\beta^4-2316288\,\alpha^8\,\beta^4+952896\,\alpha^{10}\,\beta^4-65184\,\alpha^{12}\,\beta^4-2324\,\alpha^{14}\,\beta^4+1671168\,\alpha\,\beta^5-$$

$$2097152\,\alpha^3\,\beta^5+522240\,\alpha^5\,\beta^5+1888256\,\alpha^7\,\beta^5-1419136\,\alpha^9\,\beta^5+295680\,\alpha^{11}\,\beta^5-$$

$$280\,\alpha^{13}\,\beta^5+540672\,\beta^6+245760\,\alpha^2\,\beta^6-232448\,\alpha^4\,\beta^6-901120\,\alpha^6\,\beta^6+1614528\,\alpha^8\,\beta^6-$$

$$545216\,\alpha^{10}\,\beta^6+19292\,\alpha^{12}\,\beta^6+262144\,\alpha\,\beta^7-1019904\,\alpha^3\,\beta^7+1888256\,\alpha^5\,\beta^7-$$

$$1492480\,\alpha^7\,\beta^7+638208\,\alpha^9\,\beta^7-62608\,\alpha^{11}\,\beta^7-995328\,\beta^8+1884672\,\alpha^2\,\beta^8-$$

$$2316288\,\alpha^4\,\beta^8+1614528\,\alpha^6\,\beta^8-633744\,\alpha^8\,\beta^8+114114\,\alpha^{10}\,\beta^8-1076224\,\alpha\,\beta^9+$$

$$1509376\,\alpha^3\,\beta^9-1419136\,\alpha^5\,\beta^9+638208\,\alpha^7\,\beta^9-137852\,\alpha^9\,\beta^9+182784\,\beta^{10}-$$

$$974848\,\alpha^2\,\beta^{10}+952896\,\alpha^4\,\beta^{10}-545216\,\alpha^6\,\beta^{10}+114114\,\alpha^8\,\beta^{10}+272384\,\alpha\,\beta^{11}-$$

$$521472\,\alpha^3\,\beta^{11}+295680\,\alpha^5\,\beta^{11}-62608\,\alpha^7\,\beta^{11}+71680\,\beta^{12}+48832\,\alpha^2\,\beta^{12}-65184\,\alpha^4\,\beta^{12}+$$

$$19292\,\alpha^6\,\beta^{12}+65408\,\alpha\,\beta^{13}-16128\,\alpha^3\,\beta^{13}-280\,\alpha^5\,\beta^{13}+5184\,\beta^{14}+10176\,\alpha^2\,\beta^{14}-$$

$$2324\,\alpha^4\,\beta^{14}-256\,\alpha\,\beta^{15}+784\,\alpha^3\,\beta^{15}-408\,\beta^{16}-17\,\alpha^2\,\beta^{16}-42\,\alpha\,\beta^{17}+7\,\beta^{18}\big)}\Big)\Big)\Big/$$

$$\Big(2\Big(\frac{1}{64}-\frac{\alpha^2}{32}+\frac{5\,\alpha^4}{512}+\frac{\alpha^6}{256}-\frac{3\,\alpha^8}{16384}-\frac{3\,\alpha^3\,\beta}{128}+\frac{3\,\alpha^5\,\beta}{256}+\frac{\alpha^7\,\beta}{2048}-\frac{\beta^2}{32}+\frac{7\,\alpha^2\,\beta^2}{256}-$$

$$\frac{5\,\alpha^4\,\beta^2}{256}+\frac{3\,\alpha^6\,\beta^2}{4096}-\frac{3\,\alpha\,\beta^3}{128}+\frac{\alpha^3\,\beta^3}{128}-\frac{9\,\alpha^5\,\beta^3}{2048}+\frac{5\,\beta^4}{512}-\frac{5\,\alpha^2\,\beta^4}{256}+$$

$$\frac{55\,\alpha^4\,\beta^4}{8192}+\frac{3\,\alpha\,\beta^5}{256}-\frac{9\,\alpha^3\,\beta^5}{2048}+\frac{\beta^6}{256}+\frac{3\,\alpha^2\,\beta^6}{4096}+\frac{\alpha\,\beta^7}{2048}-\frac{3\,\beta^8}{16384}\Big)\Big)\Big\},$$

$$\Big\{y \to \Big(\frac{\alpha}{128}-\frac{\alpha^3}{128}-\frac{\alpha^5}{1024}+\frac{7\,\alpha^7}{2048}-\frac{7\,\alpha^9}{32768}+\frac{\beta}{128}+\frac{\alpha^2\,\beta}{128}-\frac{13\,\alpha^4\,\beta}{1024}+\frac{13\,\alpha^6\,\beta}{2048}+$$

$$\frac{17\,\alpha^8\,\beta}{32768}+\frac{\alpha\,\beta^2}{128}+\frac{7\,\alpha^3\,\beta^2}{512}-\frac{17\,\alpha^5\,\beta^2}{2048}+$$

$$\frac{5\,\alpha^7\,\beta^2}{8192}-\frac{\beta^3}{128}+\frac{7\,\alpha^2\,\beta^3}{512}-\frac{3\,\alpha^4\,\beta^3}{2048}-$$

$$\frac{23\,\alpha^6\,\beta^3}{8192}-\frac{13\,\alpha\,\beta^4}{1024}-\frac{3\,\alpha^3\,\beta^4}{2048}+\frac{31\,\alpha^5\,\beta^4}{16384}-\frac{\beta^5}{1024}-$$

$$\frac{17\,\alpha^2\,\beta^5}{2048} + \frac{31\,\alpha^4\,\beta^5}{16\,384} + \frac{13\,\alpha\,\beta^6}{2048} - \frac{23\,\alpha^3\,\beta^6}{8192} +$$
$$\frac{7\,\beta^7}{2048} + \frac{5\,\alpha^2\,\beta^7}{8192} + \frac{17\,\alpha\,\beta^8}{32\,768} - \frac{7\,\beta^9}{32\,768} + \frac{1}{32\,768}$$
$$\Big(\sqrt{}\big(524\,288 - 1\,638\,400\,\alpha^2 + 1\,310\,720\,\alpha^4 + 540\,672\,\alpha^6 - 995\,328\,\alpha^8 + 182\,784\,\alpha^{10} + 71\,680\,\alpha^{12} +$$
$$5184\,\alpha^{14} - 408\,\alpha^{16} + 7\,\alpha^{18} + 393\,216\,\alpha\,\beta - 1\,572\,864\,\alpha^3\,\beta + 1\,671\,168\,\alpha^5\,\beta + 262\,144\,\alpha^7\,\beta -$$
$$1\,076\,224\,\alpha^9\,\beta + 272\,384\,\alpha^{11}\,\beta + 65\,408\,\alpha^{13}\,\beta - 256\,\alpha^{15}\,\beta - 42\,\alpha^{17}\,\beta - 1\,638\,400\,\beta^2 +$$
$$4\,718\,592\,\alpha^2\,\beta^2 - 4\,210\,688\,\alpha^4\,\beta^2 + 245\,760\,\alpha^6\,\beta^2 + 1\,884\,672\,\alpha^8\,\beta^2 - 974\,848\,\alpha^{10}\,\beta^2 +$$
$$48\,832\,\alpha^{12}\,\beta^2 + 10\,176\,\alpha^{14}\,\beta^2 - 17\,\alpha^{16}\,\beta^2 - 1\,572\,864\,\alpha\,\beta^3 + 3\,997\,696\,\alpha^3\,\beta^3 -$$
$$2\,097\,152\,\alpha^5\,\beta^3 - 1\,019\,904\,\alpha^7\,\beta^3 + 1\,509\,376\,\alpha^9\,\beta^3 - 521\,472\,\alpha^{11}\,\beta^3 - 16\,128\,\alpha^{13}\,\beta^3 +$$
$$784\,\alpha^{15}\,\beta^3 + 1\,310\,720\,\beta^4 - 4\,210\,688\,\alpha^2\,\beta^4 + 5\,169\,152\,\alpha^4\,\beta^4 - 232\,448\,\alpha^6\,\beta^4 -$$
$$2\,316\,288\,\alpha^8\,\beta^4 + 952\,896\,\alpha^{10}\,\beta^4 - 65\,184\,\alpha^{12}\,\beta^4 - 2324\,\alpha^{14}\,\beta^4 + 1\,671\,168\,\alpha\,\beta^5 -$$
$$2\,097\,152\,\alpha^3\,\beta^5 + 522\,240\,\alpha^5\,\beta^5 + 1\,888\,256\,\alpha^7\,\beta^5 - 1\,419\,136\,\alpha^9\,\beta^5 + 295\,680\,\alpha^{11}\,\beta^5 -$$
$$280\,\alpha^{13}\,\beta^5 + 540\,672\,\beta^6 + 245\,760\,\alpha^2\,\beta^6 - 232\,448\,\alpha^4\,\beta^6 - 901\,120\,\alpha^6\,\beta^6 + 1\,614\,528\,\alpha^8\,\beta^6 -$$
$$545\,216\,\alpha^{10}\,\beta^6 + 19\,292\,\alpha^{12}\,\beta^6 + 262\,144\,\alpha\,\beta^7 - 1\,019\,904\,\alpha^3\,\beta^7 + 1\,888\,256\,\alpha^5\,\beta^7 -$$
$$1\,492\,480\,\alpha^7\,\beta^7 + 638\,208\,\alpha^9\,\beta^7 - 62\,608\,\alpha^{11}\,\beta^7 - 995\,328\,\beta^8 + 1\,884\,672\,\alpha^2\,\beta^8 -$$
$$2\,316\,288\,\alpha^4\,\beta^8 + 1\,614\,528\,\alpha^6\,\beta^8 - 633\,744\,\alpha^8\,\beta^8 + 114\,114\,\alpha^{10}\,\beta^8 - 1\,076\,224\,\alpha\,\beta^9 +$$
$$1\,509\,376\,\alpha^3\,\beta^9 - 1\,419\,136\,\alpha^5\,\beta^9 + 638\,208\,\alpha^7\,\beta^9 - 137\,852\,\alpha^9\,\beta^9 + 182\,784\,\beta^{10} -$$
$$974\,848\,\alpha^2\,\beta^{10} + 952\,896\,\alpha^4\,\beta^{10} - 545\,216\,\alpha^6\,\beta^{10} + 114\,114\,\alpha^8\,\beta^{10} + 272\,384\,\alpha\,\beta^{11} -$$
$$521\,472\,\alpha^3\,\beta^{11} + 295\,680\,\alpha^5\,\beta^{11} - 62\,608\,\alpha^7\,\beta^{11} + 71\,680\,\beta^{12} + 48\,832\,\alpha^2\,\beta^{12} - 65\,184\,\alpha^4\,\beta^{12} +$$
$$19\,292\,\alpha^6\,\beta^{12} + 65\,408\,\alpha\,\beta^{13} - 16\,128\,\alpha^3\,\beta^{13} - 280\,\alpha^5\,\beta^{13} + 5184\,\beta^{14} + 10\,176\,\alpha^2\,\beta^{14} -$$
$$2324\,\alpha^4\,\beta^{14} - 256\,\alpha\,\beta^{15} + 784\,\alpha^3\,\beta^{15} - 408\,\beta^{16} - 17\,\alpha^2\,\beta^{16} - 42\,\alpha\,\beta^{17} + 7\,\beta^{18}\big)\Big)\Big/$$
$$\Big(2\,\Big(\frac{1}{64} - \frac{\alpha^2}{32} + \frac{5\,\alpha^4}{512} + \frac{\alpha^6}{256} - \frac{3\,\alpha^8}{16\,384} - \frac{3\,\alpha^3\,\beta}{128} + \frac{3\,\alpha^5\,\beta}{256} + \frac{\alpha^7\,\beta}{2048} - \frac{\beta^2}{32} + \frac{7\,\alpha^2\,\beta^2}{256} -$$
$$\frac{5\,\alpha^4\,\beta^2}{256} + \frac{3\,\alpha^6\,\beta^2}{4096} - \frac{3\,\alpha\,\beta^3}{128} + \frac{\alpha^3\,\beta^3}{128} - \frac{9\,\alpha^5\,\beta^3}{2048} + \frac{5\,\beta^4}{512} - \frac{5\,\alpha^2\,\beta^4}{256} +$$
$$\frac{55\,\alpha^4\,\beta^4}{8192} + \frac{3\,\alpha\,\beta^5}{256} - \frac{9\,\alpha^3\,\beta^5}{2048} + \frac{\beta^6}{256} + \frac{3\,\alpha^2\,\beta^6}{4096} + \frac{\alpha\,\beta^7}{2048} - \frac{3\,\beta^8}{16\,384}\Big)\Big)\}\}$$

(* Inserting these two outcomes of y for y1 respectively y2 in S_{7,α,β,y_j}[x] yields a septic polynomial in x *)

CoefficientList[
Liste der Koeffizienten

$$\left(-1 + x^2\right)\,(x - y1)^2\,(x - y2)^2\,(x - \alpha) - \frac{1}{2}\,(-y1 + \beta)^2\,(-y2 + \beta)^2\,(-\alpha + \beta)\,\left(-1 + \beta^2\right)\ /.$$
$$y1 \to \Big(\frac{\alpha}{128} - \frac{\alpha^3}{128} - \frac{\alpha^5}{1024} + \frac{7\,\alpha^7}{2048} - \frac{7\,\alpha^9}{32\,768} + \frac{\beta}{128} + \frac{\alpha^2\,\beta}{128} - \frac{13\,\alpha^4\,\beta}{1024} +$$
$$\frac{13\,\alpha^6\,\beta}{2048} + \frac{17\,\alpha^8\,\beta}{32\,768} + \frac{\alpha\,\beta^2}{128} + \frac{7\,\alpha^3\,\beta^2}{512} - \frac{17\,\alpha^5\,\beta^2}{2048} + \frac{5\,\alpha^7\,\beta^2}{8192} - \frac{\beta^3}{128} + \frac{7\,\alpha^2\,\beta^3}{512} -$$
$$\frac{3\,\alpha^4\,\beta^3}{2048} - \frac{23\,\alpha^6\,\beta^3}{8192} - \frac{13\,\alpha\,\beta^4}{1024} - \frac{3\,\alpha^3\,\beta^4}{2048} + \frac{31\,\alpha^5\,\beta^4}{16\,384} - \frac{\beta^5}{1024} - \frac{17\,\alpha^2\,\beta^5}{2048} +$$
$$\frac{31\,\alpha^4\,\beta^5}{16\,384} + \frac{13\,\alpha\,\beta^6}{2048} - \frac{23\,\alpha^3\,\beta^6}{8192} + \frac{7\,\beta^7}{2048} + \frac{5\,\alpha^2\,\beta^7}{8192} + \frac{17\,\alpha\,\beta^8}{32\,768} - \frac{7\,\beta^9}{32\,768} -$$

$$\frac{1}{32768}\Big(\sqrt{}\big(524288 - 1638400\,\alpha^2 + 1310720\,\alpha^4 + 540672\,\alpha^6 - 995328\,\alpha^8 + 182784\,\alpha^{10} +$$
$$71680\,\alpha^{12} + 5184\,\alpha^{14} - 408\,\alpha^{16} + 7\,\alpha^{18} + 393216\,\alpha\,\beta - 1572864\,\alpha^3\,\beta + 1671168\,\alpha^5\,\beta +$$
$$262144\,\alpha^7\,\beta - 1076224\,\alpha^9\,\beta + 272384\,\alpha^{11}\,\beta + 65408\,\alpha^{13}\,\beta - 256\,\alpha^{15}\,\beta - 42\,\alpha^{17}\,\beta -$$
$$1638400\,\beta^2 + 4718592\,\alpha^2\,\beta^2 - 4210688\,\alpha^4\,\beta^2 + 245760\,\alpha^6\,\beta^2 + 1884672\,\alpha^8\,\beta^2 -$$
$$974848\,\alpha^{10}\,\beta^2 + 48832\,\alpha^{12}\,\beta^2 + 10176\,\alpha^{14}\,\beta^2 - 17\,\alpha^{16}\,\beta^2 - 1572864\,\alpha\,\beta^3 +$$
$$3997696\,\alpha^3\,\beta^3 - 2097152\,\alpha^5\,\beta^3 - 1019904\,\alpha^7\,\beta^3 + 1509376\,\alpha^9\,\beta^3 -$$
$$521472\,\alpha^{11}\,\beta^3 - 16128\,\alpha^{13}\,\beta^3 + 784\,\alpha^{15}\,\beta^3 + 1310720\,\beta^4 - 4210688\,\alpha^2\,\beta^4 +$$
$$5169152\,\alpha^4\,\beta^4 - 232448\,\alpha^6\,\beta^4 - 2316288\,\alpha^8\,\beta^4 + 952896\,\alpha^{10}\,\beta^4 - 65184\,\alpha^{12}\,\beta^4 -$$
$$2324\,\alpha^{14}\,\beta^4 + 1671168\,\alpha\,\beta^5 - 2097152\,\alpha^3\,\beta^5 + 522240\,\alpha^5\,\beta^5 + 1888256\,\alpha^7\,\beta^5 -$$
$$1419136\,\alpha^9\,\beta^5 + 295680\,\alpha^{11}\,\beta^5 - 280\,\alpha^{13}\,\beta^5 + 540672\,\beta^6 + 245760\,\alpha^2\,\beta^6 -$$
$$232448\,\alpha^4\,\beta^6 - 901120\,\alpha^6\,\beta^6 + 1614528\,\alpha^8\,\beta^6 - 545216\,\alpha^{10}\,\beta^6 + 19292\,\alpha^{12}\,\beta^6 +$$
$$262144\,\alpha\,\beta^7 - 1019904\,\alpha^3\,\beta^7 + 1888256\,\alpha^5\,\beta^7 - 1492480\,\alpha^7\,\beta^7 + 638208\,\alpha^9\,\beta^7 -$$
$$62608\,\alpha^{11}\,\beta^7 - 995328\,\beta^8 + 1884672\,\alpha^2\,\beta^8 - 2316288\,\alpha^4\,\beta^8 + 1614528\,\alpha^6\,\beta^8 -$$
$$633744\,\alpha^8\,\beta^8 + 114114\,\alpha^{10}\,\beta^8 - 1076224\,\alpha\,\beta^9 + 1509376\,\alpha^3\,\beta^9 - 1419136\,\alpha^5\,\beta^9 +$$
$$638208\,\alpha^7\,\beta^9 - 137852\,\alpha^9\,\beta^9 + 182784\,\beta^{10} - 974848\,\alpha^2\,\beta^{10} + 952896\,\alpha^4\,\beta^{10} -$$
$$545216\,\alpha^6\,\beta^{10} + 114114\,\alpha^8\,\beta^{10} + 272384\,\alpha\,\beta^{11} - 521472\,\alpha^3\,\beta^{11} + 295680\,\alpha^5\,\beta^{11} -$$
$$62608\,\alpha^7\,\beta^{11} + 71680\,\beta^{12} + 48832\,\alpha^2\,\beta^{12} - 65184\,\alpha^4\,\beta^{12} + 19292\,\alpha^6\,\beta^{12} +$$
$$65408\,\alpha\,\beta^{13} - 16128\,\alpha^3\,\beta^{13} - 280\,\alpha^5\,\beta^{13} + 5184\,\beta^{14} + 10176\,\alpha^2\,\beta^{14} -$$
$$2324\,\alpha^4\,\beta^{14} - 256\,\alpha\,\beta^{15} + 784\,\alpha^3\,\beta^{15} - 408\,\beta^{16} - 17\,\alpha^2\,\beta^{16} - 42\,\alpha\,\beta^{17} + 7\,\beta^{18}\big)\Big)\Big/$$

$$\Big(2\Big(\frac{1}{64} - \frac{\alpha^2}{32} + \frac{5\,\alpha^4}{512} + \frac{\alpha^6}{256} - \frac{3\,\alpha^8}{16384} - \frac{3\,\alpha^3\,\beta}{128} + \frac{3\,\alpha^5\,\beta}{256} + \frac{\alpha^7\,\beta}{2048} - \frac{\beta^2}{32} + \frac{7\,\alpha^2\,\beta^2}{256} -$$
$$\frac{5\,\alpha^4\,\beta^2}{256} + \frac{3\,\alpha^6\,\beta^2}{4096} - \frac{3\,\alpha\,\beta^3}{128} + \frac{\alpha^3\,\beta^3}{128} - \frac{9\,\alpha^5\,\beta^3}{2048} + \frac{5\,\beta^4}{512} - \frac{5\,\alpha^2\,\beta^4}{256} +$$
$$\frac{55\,\alpha^4\,\beta^4}{8192} + \frac{3\,\alpha\,\beta^5}{256} - \frac{9\,\alpha^3\,\beta^5}{2048} + \frac{\beta^6}{256} + \frac{3\,\alpha^2\,\beta^6}{4096} + \frac{\alpha\,\beta^7}{2048} - \frac{3\,\beta^8}{16384}\Big)\Big)\ /.$$

$$y2 \to \Big(\frac{\alpha}{128} - \frac{\alpha^3}{128} - \frac{\alpha^5}{1024} + \frac{7\,\alpha^7}{2048} - \frac{7\,\alpha^9}{32768} + \frac{\beta}{128} + \frac{\alpha^2\,\beta}{128} - \frac{13\,\alpha^4\,\beta}{1024} + \frac{13\,\alpha^6\,\beta}{2048} +$$
$$\frac{17\,\alpha^8\,\beta}{32768} + \frac{\alpha\,\beta^2}{128} + \frac{7\,\alpha^3\,\beta^2}{512} - \frac{17\,\alpha^5\,\beta^2}{2048} + \frac{5\,\alpha^7\,\beta^2}{8192} - \frac{\beta^3}{128} +$$
$$\frac{7\,\alpha^2\,\beta^3}{512} - \frac{3\,\alpha^4\,\beta^3}{2048} - \frac{23\,\alpha^6\,\beta^3}{8192} - \frac{13\,\alpha\,\beta^4}{1024} - \frac{3\,\alpha^3\,\beta^4}{2048} +$$
$$\frac{31\,\alpha^5\,\beta^4}{16384} - \frac{\beta^5}{1024} - \frac{17\,\alpha^2\,\beta^5}{2048} + \frac{31\,\alpha^4\,\beta^5}{16384} + \frac{13\,\alpha\,\beta^6}{2048} -$$
$$\frac{23\,\alpha^3\,\beta^6}{8192} + \frac{7\,\beta^7}{2048} + \frac{5\,\alpha^2\,\beta^7}{8192} + \frac{17\,\alpha\,\beta^8}{32768} - \frac{7\,\beta^9}{32768} +$$
$$\frac{1}{32768}\Big(\sqrt{}\big(524288 - 1638400\,\alpha^2 + 1310720\,\alpha^4 + 540672\,\alpha^6 - 995328\,\alpha^8 + 182784\,\alpha^{10} +$$
$$71680\,\alpha^{12} + 5184\,\alpha^{14} - 408\,\alpha^{16} + 7\,\alpha^{18} + 393216\,\alpha\,\beta - 1572864\,\alpha^3\,\beta +$$
$$1671168\,\alpha^5\,\beta + 262144\,\alpha^7\,\beta - 1076224\,\alpha^9\,\beta + 272384\,\alpha^{11}\,\beta + 65408\,\alpha^{13}\,\beta -$$
$$256\,\alpha^{15}\,\beta - 42\,\alpha^{17}\,\beta - 1638400\,\beta^2 + 4718592\,\alpha^2\,\beta^2 - 4210688\,\alpha^4\,\beta^2 +$$
$$245760\,\alpha^6\,\beta^2 + 1884672\,\alpha^8\,\beta^2 - 974848\,\alpha^{10}\,\beta^2 + 48832\,\alpha^{12}\,\beta^2 + 10176\,\alpha^{14}\,\beta^2 -$$
$$17\,\alpha^{16}\,\beta^2 - 1572864\,\alpha\,\beta^3 + 3997696\,\alpha^3\,\beta^3 - 2097152\,\alpha^5\,\beta^3 - 1019904\,\alpha^7\,\beta^3 +$$
$$1509376\,\alpha^9\,\beta^3 - 521472\,\alpha^{11}\,\beta^3 - 16128\,\alpha^{13}\,\beta^3 + 784\,\alpha^{15}\,\beta^3 + 1310720\,\beta^4 -$$

$$4\,210\,688\,\alpha^2\,\beta^4 + 5\,169\,152\,\alpha^4\,\beta^4 - 232\,448\,\alpha^6\,\beta^4 - 2\,316\,288\,\alpha^8\,\beta^4 + 952\,896\,\alpha^{10}\,\beta^4 -$$
$$65\,184\,\alpha^{12}\,\beta^4 - 2324\,\alpha^{14}\,\beta^4 + 1\,671\,168\,\alpha\,\beta^5 - 2\,097\,152\,\alpha^3\,\beta^5 + 522\,240\,\alpha^5\,\beta^5 +$$
$$1\,888\,256\,\alpha^7\,\beta^5 - 1\,419\,136\,\alpha^9\,\beta^5 + 295\,680\,\alpha^{11}\,\beta^5 - 280\,\alpha^{13}\,\beta^5 + 540\,672\,\beta^6 +$$
$$245\,760\,\alpha^2\,\beta^6 - 232\,448\,\alpha^4\,\beta^6 - 901\,120\,\alpha^6\,\beta^6 + 1\,614\,528\,\alpha^8\,\beta^6 - 545\,216\,\alpha^{10}\,\beta^6 +$$
$$19\,292\,\alpha^{12}\,\beta^6 + 262\,144\,\alpha\,\beta^7 - 1\,019\,904\,\alpha^3\,\beta^7 + 1\,888\,256\,\alpha^5\,\beta^7 - 1\,492\,480\,\alpha^7\,\beta^7 +$$
$$638\,208\,\alpha^9\,\beta^7 - 62\,608\,\alpha^{11}\,\beta^7 - 995\,328\,\beta^8 + 1\,884\,672\,\alpha^2\,\beta^8 - 2\,316\,288\,\alpha^4\,\beta^8 +$$
$$1\,614\,528\,\alpha^6\,\beta^8 - 633\,744\,\alpha^8\,\beta^8 + 114\,114\,\alpha^{10}\,\beta^8 - 1\,076\,224\,\alpha\,\beta^9 + 1\,509\,376\,\alpha^3\,\beta^9 -$$
$$1\,419\,136\,\alpha^5\,\beta^9 + 638\,208\,\alpha^7\,\beta^9 - 137\,852\,\alpha^9\,\beta^9 + 182\,784\,\beta^{10} - 974\,848\,\alpha^2\,\beta^{10} +$$
$$952\,896\,\alpha^4\,\beta^{10} - 545\,216\,\alpha^6\,\beta^{10} + 114\,114\,\alpha^8\,\beta^{10} + 272\,384\,\alpha\,\beta^{11} - 521\,472\,\alpha^3\,\beta^{11} +$$
$$295\,680\,\alpha^5\,\beta^{11} - 62\,608\,\alpha^7\,\beta^{11} + 71\,680\,\beta^{12} + 48\,832\,\alpha^2\,\beta^{12} - 65\,184\,\alpha^4\,\beta^{12} +$$
$$19\,292\,\alpha^6\,\beta^{12} + 65\,408\,\alpha\,\beta^{13} - 16\,128\,\alpha^3\,\beta^{13} - 280\,\alpha^5\,\beta^{13} + 5184\,\beta^{14} + 10\,176\,\alpha^2\,\beta^{14} -$$
$$2324\,\alpha^4\,\beta^{14} - 256\,\alpha\,\beta^{15} + 784\,\alpha^3\,\beta^{15} - 408\,\beta^{16} - 17\,\alpha^2\,\beta^{16} - 42\,\alpha\,\beta^{17} + 7\,\beta^{18}))\Big) \Big/$$
$$\Big(2\,\Big(\frac{1}{64} - \frac{\alpha^2}{32} + \frac{5\,\alpha^4}{512} + \frac{\alpha^6}{256} - \frac{3\,\alpha^8}{16\,384} - \frac{3\,\alpha^3\,\beta}{128} + \frac{3\,\alpha^5\,\beta}{256} + \frac{\alpha^7\,\beta}{2048} - \frac{\beta^2}{32} + \frac{7\,\alpha^2\,\beta^2}{256} -$$
$$\frac{5\,\alpha^4\,\beta^2}{256} + \frac{3\,\alpha^6\,\beta^2}{4096} - \frac{3\,\alpha\,\beta^3}{128} + \frac{\alpha^3\,\beta^3}{128} - \frac{9\,\alpha^5\,\beta^3}{2048} + \frac{5\,\beta^4}{512} - \frac{5\,\alpha^2\,\beta^4}{256} + \frac{55\,\alpha^4\,\beta^4}{8192} +$$
$$\frac{3\,\alpha\,\beta^5}{256} - \frac{9\,\alpha^3\,\beta^5}{2048} + \frac{\beta^6}{256} + \frac{3\,\alpha^2\,\beta^6}{4096} + \frac{\alpha\,\beta^7}{2048} - \frac{3\,\beta^8}{16\,384}\Big)\Big),\ x\Big]$$ `// FullSimplify`

vereinfache vollständig

Out[*]=

$$\Big\{\Big(49\,\alpha^{20}\,\beta\,(5 - 13\,\beta^2) + 49\,\alpha^{21}\,(1 + \beta^2) + 14\,\alpha^{19}\,(-44 - 281\,\beta^2 + 255\,\beta^4) -$$
$$14\,\alpha^{18}\,\beta\,(356 - 1679\,\beta^2 + 759\,\beta^4) + \alpha^{16}\,\beta\,(53\,456 - 260\,248\,\beta^2 + 130\,701\,\beta^4 + 12\,699\,\beta^6) +$$
$$\alpha^{17}\,(12\,240 + 66\,808\,\beta^2 - 84\,111\,\beta^4 + 14\,289\,\beta^6) -$$
$$8\,\alpha^{15}\,(8768 + 74\,160\,\beta^2 - 63\,964\,\beta^4 - 22\,935\,\beta^6 + 11\,985\,\beta^8) +$$
$$8\,\alpha^{14}\,\beta\,(-51\,136 + 96\,016\,\beta^2 + 135\,668\,\beta^4 - 155\,289\,\beta^6 + 24\,225\,\beta^8) +$$
$$\alpha^{13}\,(557\,568 + 309\,248\,\beta^2 + 5\,159\,360\,\beta^4 - 7\,379\,744\,\beta^6 + 2\,353\,218\,\beta^8 - 173\,502\,\beta^{10}) +$$
$$2\,\alpha^{12}\,\beta\,(367\,872 + 4\,472\,832\,\beta^2 - 10\,093\,088\,\beta^4 + 5\,940\,304\,\beta^6 - 798\,315\,\beta^8 - 35\,581\,\beta^{10}) + 4\,\alpha^{10}\,\beta$$
$$(228\,352 - 7\,639\,296\,\beta^2 + 8\,253\,312\,\beta^4 + 1\,263\,712\,\beta^6 - 3\,863\,908\,\beta^8 + 1\,352\,963\,\beta^{10} - 178\,347\,\beta^{12}) +$$
$$4\,\alpha^{11}\,(-109\,568 + 1\,462\,016\,\beta^2 - 7\,297\,920\,\beta^4 + 6\,212\,832\,\beta^6 - 562\,988\,\beta^8 -$$
$$433\,237\,\beta^{10} + 112\,931\,\beta^{12}) + 2\,\alpha^9\,(-913\,408 - 6\,551\,552\,\beta^2 + 12\,086\,016\,\beta^4 +$$
$$12\,306\,944\,\beta^6 - 26\,150\,672\,\beta^8 + 12\,759\,176\,\beta^{10} - 3\,393\,947\,\beta^{12} + 341\,445\,\beta^{14}) -$$
$$8\,\alpha^6\,\beta\,(-212\,992 - 6\,180\,864\,\beta^2 + 24\,314\,880\,\beta^4 - 30\,825\,728\,\beta^6 + 19\,685\,440\,\beta^8 -$$
$$8\,710\,384\,\beta^{10} + 2\,152\,828\,\beta^{12} - 247\,359\,\beta^{14} + 3927\,\beta^{16}) +$$
$$8\,\alpha^7\,(311\,296 + 4096\,\beta^2 + 8\,803\,328\,\beta^4 - 22\,584\,064\,\beta^6 + 21\,324\,352\,\beta^8 -$$
$$10\,382\,672\,\beta^{10} + 3\,282\,060\,\beta^{12} - 475\,809\,\beta^{14} + 21\,879\,\beta^{16}) +$$
$$\alpha^5\,(851\,968 + 21\,168\,128\,\beta^2 - 147\,865\,600\,\beta^4 + 263\,495\,680\,\beta^6 - 232\,813\,056\,\beta^8 +$$
$$132\,206\,592\,\beta^{10} - 44\,855\,360\,\beta^{12} + 7\,820\,640\,\beta^{14} - 669\,459\,\beta^{16} - 8211\,\beta^{18}) +$$
$$\alpha^4\,\beta\,(3\,211\,264 - 64\,946\,176\,\beta^2 + 176\,406\,528\,\beta^4 - 221\,585\,408\,\beta^6 + 172\,045\,824\,\beta^8 -$$
$$83\,879\,936\,\beta^{10} + 21\,634\,496\,\beta^{12} - 3\,261\,920\,\beta^{14} + 49\,665\,\beta^{16} + 6615\,\beta^{18}) -$$
$$2\,\alpha^2\,\beta\,(1\,835\,008 - 14\,745\,600\,\beta^2 + 38\,404\,096\,\beta^4 - 57\,294\,848\,\beta^6 + 48\,424\,960\,\beta^8 -$$
$$22\,564\,352\,\beta^{10} + 7\,184\,128\,\beta^{12} - 1\,194\,816\,\beta^{14} - 76\,452\,\beta^{16} + 10\,631\,\beta^{18} + 17\,\beta^{20}) -$$
$$2\,\alpha^3\,(1\,310\,720 + 8\,847\,360\,\beta^2 - 48\,365\,568\,\beta^4 + 84\,197\,376\,\beta^6 - 83\,101\,696\,\beta^8 +$$
$$50\,762\,240\,\beta^{10} - 17\,438\,464\,\beta^{12} + 4\,297\,152\,\beta^{14} - 478\,700\,\beta^{16} - 31\,105\,\beta^{18} + 727\,\beta^{20}) +$$

$$\alpha\ (1048576 + 3670016\,\beta^2 - 23003136\,\beta^4 + 44236800\,\beta^6 - 42655744\,\beta^8 + 27217920\,\beta^{10} -$$
$$14689792\,\beta^{12} + 4301824\,\beta^{14} - 816\,\beta^{16} - 118856\,\beta^{18} + 101\,\beta^{20} + 69\,\beta^{22}) +$$
$$2\,\alpha^8\,\beta\ (-1798144 + 5724160\,\beta^2 + 31098624\,\beta^4 - 61881856\,\beta^6 +$$
$$3\,\beta^8\ (13356624 - 4714232\,\beta^2 + 958411\,\beta^4 - 72403\,\beta^6)) -$$
$$(-1 + \beta)\ \beta\ (1 + \beta)\ (1024 + 3\,\beta^2\ (-768 + 896\,\beta^2 - 224\,\beta^4 - 36\,\beta^6 + \beta^8))^2) \Big/$$
$$(128\ (-4 + \alpha^2 + (-4 + \beta)\ \beta - 2\,\alpha\ (2 + \beta))^2\ (-4 + \alpha^2 - 2\,\alpha\ (-2 + \beta) + \beta\ (4 + \beta))^2$$
$$(-4 + (\alpha - \beta)\ (3\,\alpha + \beta))^2\ (4 + (\alpha - \beta)\ (\alpha + 3\,\beta))^2),$$
$$-(((1024 + 7\,\alpha^{10} - 14\,\alpha^9\,\beta - 1280\,\beta^2 - 384\,\beta^4 + 736\,\beta^6 - 44\,\beta^8 + 7\,\beta^{10} + 8\,\alpha^7\,\beta\ (28 - 5\,\beta^2) -$$
$$\alpha^8\ (44 + 5\,\beta^2) + \alpha^6\ (736 - 464\,\beta^2 + 254\,\beta^4) + \alpha^5\ (-64\,\beta + 544\,\beta^3 - 404\,\beta^5) -$$
$$8\,\alpha^3\,\beta\ (64 + 112\,\beta^2 - 68\,\beta^4 + 5\,\beta^6) + \alpha^4\ (-384 - 224\,\beta^2 - 520\,\beta^4 + 254\,\beta^6) -$$
$$\alpha^2\ (1280 - 1792\,\beta^2 + 224\,\beta^4 + 464\,\beta^6 + 5\,\beta^8) - 2\,\alpha\,\beta\ (-256 + 256\,\beta^2 + 32\,\beta^4 - 112\,\beta^6 + 7\,\beta^8))$$
$$(1024 + 63\,\alpha^{10} - 150\,\alpha^9\,\beta - 1280\,\beta^2 - 384\,\beta^4 + 736\,\beta^6 - 44\,\beta^8 + 7\,\beta^{10} + 24\,\alpha^7\,\beta\ (-60 + 29\,\beta^2) -$$
$$5\,\alpha^8\ (188 + 33\,\beta^2) + \alpha^6\ (992 + 1712\,\beta^2 - 242\,\beta^4) + 4\,\alpha^5\,\beta\ (816 + 232\,\beta^2 - 225\,\beta^4) -$$
$$40\,\alpha^3\,\beta\ (64 + 112\,\beta^2 - 68\,\beta^4 + 5\,\beta^6) + 2\,\alpha^4\ (832 - 1904\,\beta^2 - 68\,\beta^4 + 495\,\beta^6) + 6\,\alpha\,\beta\ (-256 +$$
$$256\,\beta^2 + 32\,\beta^4 - 112\,\beta^6 + 7\,\beta^8) - \alpha^2\ (3328 + 256\,\beta^2 - 3104\,\beta^4 + 2128\,\beta^6 + 141\,\beta^8))) \Big/$$
$$(64\ (-4 + \alpha^2 + (-4 + \beta)\ \beta - 2\,\alpha\ (2 + \beta))^2\ (-4 + \alpha^2 - 2\,\alpha\ (-2 + \beta) + \beta\ (4 + \beta))^2$$
$$(-4 + (\alpha - \beta)\ (3\,\alpha + \beta))^2\ (4 + (\alpha - \beta)\ (\alpha + 3\,\beta))^2)),$$
$$(-49\,\alpha^{21} + 196\,\alpha^{20}\,\beta + 14\,\alpha^{19}\ (152 - 9\,\beta^2) + 140\,\alpha^{18}\,\beta\ (-82 + 3\,\beta^2) +$$
$$\alpha^{17}\ (-55344 + 13184\,\beta^2 - 4701\,\beta^4) + 8\,\alpha^{16}\,\beta\ (10524 + 3113\,\beta^2 + 1546\,\beta^4) +$$
$$16\,\alpha^{14}\,\beta\ (9888 - 608\,\beta^2 - 19406\,\beta^4 + 1497\,\beta^6) -$$
$$8\,\alpha^{15}\ (-52864 - 24816\,\beta^2 + 360\,\beta^4 + 1741\,\beta^6) +$$
$$8\,\alpha^{12}\,\beta\ (-765952 + 202944\,\beta^2 + 77296\,\beta^4 - 48796\,\beta^6 + 28151\,\beta^8) -$$
$$2\,\alpha^{13}\ (947456 + 466944\,\beta^2 + 636320\,\beta^4 - 351296\,\beta^6 + 47673\,\beta^8) +$$
$$8\,\beta\ (-256 + 256\,\beta^2 + 32\,\beta^4 - 112\,\beta^6 + 7\,\beta^8)\ (1024 - 1280\,\beta^2 - 384\,\beta^4 + 736\,\beta^6 - 44\,\beta^8 + 7\,\beta^{10}) -$$
$$\alpha\ (1024 - 1280\,\beta^2 - 384\,\beta^4 + 736\,\beta^6 - 44\,\beta^8 + 7\,\beta^{10})$$
$$(7168 - 7424\,\beta^2 - 1152\,\beta^4 + 3424\,\beta^6 + 44\,\beta^8 + 7\,\beta^{10}) +$$
$$8\,\alpha^{10}\,\beta\ (1730048 + 125184\,\beta^2 + 931136\,\beta^4 - 819968\,\beta^6 + 115654\,\beta^8 + 28151\,\beta^{10}) -$$
$$4\,\alpha^{11}\ (526336 - 2413312\,\beta^2 + 1089792\,\beta^4 - 874528\,\beta^6 + 152712\,\beta^8 + 73997\,\beta^{10}) + 2\,\alpha^9$$
$$(7131136 - 6995968\,\beta^2 + 10482944\,\beta^4 - 9389056\,\beta^6 + 3219056\,\beta^8 + 35904\,\beta^{10} - 47673\,\beta^{12}) +$$
$$16\,\alpha^8\,\beta\ (181248 - 484608\,\beta^2 - 1935232\,\beta^4 + 1689952\,\beta^6 - 285116\,\beta^8 - 66089\,\beta^{10} + 1497\,\beta^{12}) +$$
$$16\,\alpha^6\,\beta\ (-1892352 + 1179648\,\beta^2 + 1869312\,\beta^4 -$$
$$2100992\,\beta^6 + 159968\,\beta^8 + 161440\,\beta^{10} - 15366\,\beta^{12} + 773\,\beta^{14}) -$$
$$8\,\alpha^7\ (1146880 + 1773568\,\beta^2 + 759808\,\beta^4 - 2241792\,\beta^6 + 1749632\,\beta^8 - 81296\,\beta^{10} -$$
$$114328\,\beta^{12} + 1741\,\beta^{14}) + 4\,\alpha^4\,\beta\ (5963776 - 5144576\,\beta^2 - 843776\,\beta^4 +$$
$$964608\,\beta^6 + 1561600\,\beta^8 - 321408\,\beta^{10} - 17056\,\beta^{12} + 6440\,\beta^{14} + 105\,\beta^{16}) -$$
$$\alpha^5\ (16056320 - 51380224\,\beta^2 + 34095104\,\beta^4 + 9011200\,\beta^6 - 16402944\,\beta^8 +$$
$$831488\,\beta^{10} + 1812288\,\beta^{12} + 60544\,\beta^{14} + 4701\,\beta^{16}) +$$
$$2\,\alpha^3\ (11010048 - 23003136\,\beta^2 + 16908288\,\beta^4 + 1163264\,\beta^6 - 7344128\,\beta^8 +$$
$$2745856\,\beta^{10} + 299520\,\beta^{12} + 123328\,\beta^{14} + 9256\,\beta^{16} - 63\,\beta^{18}) +$$
$$4\,\alpha^2\,\beta\ (-393216 + 1900544\,\beta^2 - 2129920\,\beta^4 + 229376\,\beta^6 + 957440\,\beta^8 -$$
$$480768\,\beta^{10} + 30592\,\beta^{12} + 14592\,\beta^{14} - 2310\,\beta^{16} + 49\,\beta^{18})) \Big/$$

$$\left(64\left(-4+\alpha^2+(-4+\beta)\beta-2\alpha(2+\beta)\right)^2\left(-4+\alpha^2-2\alpha(-2+\beta)+\beta(4+\beta)\right)^2\right.$$
$$\left.(-4+(\alpha-\beta)(3\alpha+\beta))^2(4+(\alpha-\beta)(\alpha+3\beta))^2\right),$$

$$\Big(5\,242\,880+441\alpha^{20}-1932\alpha^{19}\beta-17\,301\,504\beta^2+14\,483\,456\beta^4+5\,636\,096\beta^6-$$
$$11\,558\,912\beta^8+2\,461\,696\beta^{10}+1\,217\,024\beta^{12}-340\,480\beta^{14}+46\,608\beta^{16}-$$
$$1736\beta^{18}+49\beta^{20}+14\alpha^{18}\left(-844+45\beta^2\right)+4\alpha^{17}\beta\left(9004+1353\beta^2\right)-$$
$$16\alpha^{15}\beta\left(17\,328+3160\beta^2+3959\beta^4\right)+\alpha^{16}\left(179\,216-18\,568\beta^2+11\,389\beta^4\right)+$$
$$8\alpha^{14}\left(-176\,448-141\,392\beta^2+20\,508\beta^4+3449\beta^6\right)+$$
$$2\alpha^{12}\left(139\,520+2\,602\,240\beta^2+515\,040\beta^4+70\,640\beta^6-173\,007\beta^8\right)+$$
$$8\alpha^{11}\beta\left(1\,575\,680+764\,160\beta^2-511\,584\beta^4+24\,496\beta^6+2011\beta^8\right)-$$
$$8\alpha^9\beta\left(-74\,752+4\,707\,584\beta^2-1\,036\,160\beta^4-662\,112\beta^6+82\,748\beta^8+83\,321\beta^{10}\right)+$$
$$4\alpha^{10}\left(2\,593\,792-1\,147\,648\beta^2-692\,864\beta^4-106\,336\beta^6+47\,860\beta^8+134\,321\beta^{10}\right)-$$
$$16\alpha^7\beta\left(2\,650\,112-2\,783\,232\beta^2-1\,680\,128\beta^4+2\,472\,192\beta^6-424\,048\beta^8-90\,392\beta^{10}+643\beta^{12}\right)+$$
$$2\alpha^8\left(-8\,204\,288-12\,408\,832\beta^2+6\,526\,720\beta^4+740\,608\beta^6-2\,851\,536\beta^8-\right.$$
$$\left.125\,304\beta^{10}+160\,521\beta^{12}\right)-16\alpha^5\beta\left(-3\,162\,112+577\,536\beta^2+\right.$$
$$\left.3\,486\,720\beta^4-3\,776\,256\beta^6+690\,752\beta^8+87\,536\beta^{10}-25\,876\beta^{12}+355\beta^{14}\right)-$$
$$8\alpha^6\left(802\,816-6\,975\,488\beta^2+2\,108\,416\beta^4+3\,083\,520\beta^6-3\,703\,360\beta^8+625\,968\beta^{10}+\right.$$
$$\left.156\,788\beta^{12}+4631\beta^{14}\right)+4\alpha^3\beta\left(-2\,818\,048+393\,216\beta^2+9\,814\,016\beta^4-\right.$$
$$\left.7\,938\,048\beta^6-25\,088\beta^8+1\,044\,992\beta^{10}-218\,688\beta^{12}-6240\beta^{14}+21\beta^{16}\right)-$$
$$2\alpha^2\left(4+\beta^2\right)\left(2\,949\,120-6\,029\,312\beta^2+2\,670\,592\beta^4+2\,744\,320\beta^6-2\,988\,544\beta^8+\right.$$
$$\left.618\,496\beta^{10}+248\,768\beta^{12}-10\,016\beta^{14}+805\beta^{16}\right)+$$
$$\alpha^4\left(31\,784\,960-52\,822\,016\beta^2+21\,479\,424\beta^4+54\,468\,608\beta^6-52\,380\,160\beta^8+\right.$$
$$\left.9\,186\,816\beta^{10}+3\,564\,480\beta^{12}-21\,920\beta^{14}+10\,533\beta^{16}\right)+$$
$$4\alpha\beta\left(-1\,310\,720+589\,824\beta^2+2\,686\,976\beta^4-2\,408\,448\beta^6-407\,552\beta^8+\right.$$
$$\left.978\,432\beta^{10}-360\,192\beta^{12}+16\,960\beta^{14}-84\beta^{16}+49\beta^{18}\right)+$$
$$16\alpha^{13}\beta\left(-267\,712+3\beta^2\left(49\,168-6372\beta^2+4203\beta^4\right)\right)\Big)\Big/$$
$$\left(64\left(-4+\alpha^2+(-4+\beta)\beta-2\alpha(2+\beta)\right)^2\left(-4+\alpha^2-2\alpha(-2+\beta)+\beta(4+\beta)\right)^2\right.$$
$$\left.(-4+(\alpha-\beta)(3\alpha+\beta))^2(4+(\alpha-\beta)(\alpha+3\beta))^2\right),$$

$$\frac{\alpha\left(-321\,516+6289\alpha^2\right)}{1944}-\frac{7}{216}\left(356+179\alpha^2\right)\beta-$$
$$\frac{553\alpha\beta^2}{216}-$$
$$\frac{49\beta^3}{72}+$$
$$\frac{(1+\alpha)(-308+\alpha(-526+\alpha(-161+31\alpha-73\beta)-305\beta)-234\beta)}{-4+\alpha^2+(-4+\beta)\beta-2\alpha(2+\beta)}+$$
$$\frac{16\left(-1+\alpha^2\right)^2(\alpha(-4+3\alpha(\alpha-\beta))+\beta)}{(-4+(\alpha-\beta)(3\alpha+\beta))^2}+$$
$$\frac{16(-1+\alpha)^2(34-41\beta+\alpha(-50+\alpha(8+4\alpha-7\beta)+40\beta))}{\left(-4+\alpha^2-2\alpha(-2+\beta)+\beta(4+\beta)\right)^2}-$$

$$\frac{(-1+\alpha)\ (308-234\,\beta+\alpha\ (-526+\alpha\ (161+31\,\alpha-73\,\beta)\ +305\,\beta))}{-4+\alpha^2-2\,\alpha\ (-2+\beta)\ +\beta\ (4+\beta)}+$$

$$\frac{\left(-1+\alpha^2\right)\ \left(18\,\beta+\alpha\ \left(-44+31\,\alpha^2-19\,\alpha\,\beta\right)\right)}{4-3\,\alpha^2+2\,\alpha\,\beta+\beta^2}+$$

$$\frac{16\ (1+\alpha)^2\ (-34-41\,\beta+\alpha\ (\alpha\ (-8+4\,\alpha-7\,\beta)\ -10\ (5+4\,\beta)))}{\left(-4+\alpha^2+\ (-4+\beta)\ \beta-2\,\alpha\ (2+\beta)\right)^2}+$$

$$\frac{-1458\,\beta+\alpha\ \left(-3456+\alpha\ \left(-837\,\beta+\alpha\ \left(2283-515\,\alpha^2+543\,\alpha\,\beta\right)\right)\right)}{729\ (4+\ (\alpha-\beta)\ (\alpha+3\,\beta))}+$$

$$\frac{432\,\beta-16\,\alpha\ (-54+\alpha\ (27\,\beta+\alpha\ (111+\alpha\ (\alpha\ (-32+7\,\alpha\ (\alpha-\beta))\ +39\,\beta))))}{729\ (4+\ (\alpha-\beta)\ (\alpha+3\,\beta))^2},$$

$$\frac{7}{54}\ \left(172+65\,\alpha^2\right)\ +$$

$$\frac{91\,\alpha\,\beta}{27}+$$

$$\frac{35\,\beta^2}{18}+$$

$$\frac{\left(-1+\alpha^2\right)\ \left(-26+21\,\alpha^2-17\,\alpha\,\beta\right)}{4-3\,\alpha^2+2\,\alpha\,\beta+\beta^2}-$$

$$\frac{(1+\alpha)\ (-80+\alpha\ (-79+11\,\alpha-41\,\beta)\ -75\,\beta)}{-4+\alpha^2+\ (-4+\beta)\ \beta-2\,\alpha\ (2+\beta)}-$$

$$\frac{16\ (-1+\alpha)^2\ (-5+\alpha\ (4+\alpha-2\,\beta)\ +6\,\beta)}{\left(-4+\alpha^2-2\,\alpha\ (-2+\beta)\ +\beta\ (4+\beta)\right)^2}+$$

$$\frac{(-1+\alpha)\ (-80+\alpha\ (79+11\,\alpha-41\,\beta)\ +75\,\beta)}{-4+\alpha^2-2\,\alpha\ (-2+\beta)\ +\beta\ (4+\beta)}+$$

$$\frac{16\ \left(-1+\alpha^2\right)^2\ \left(-1+\alpha^2-\alpha\,\beta\right)}{(-4+\ (\alpha-\beta)\ (3\,\alpha+\beta))^2}+$$

$$\frac{16\ (1+\alpha)^2\ (5+6\,\beta+\alpha\ (4-\alpha+2\,\beta))}{\left(-4+\alpha^2+\ (-4+\beta)\ \beta-2\,\alpha\ (2+\beta)\right)^2}+$$

$$\frac{162-1071\,\alpha^2+245\,\alpha^4-3\,\alpha\ \left(117+83\,\alpha^2\right)\ \beta}{243\ (4+\ (\alpha-\beta)\ (\alpha+3\,\beta))}+$$

$$\frac{16\ \left(9+\alpha\ \left(27\,\beta+\alpha\ \left(27+\alpha\ \left(6\,\beta+\alpha\ \left(-5+\alpha^2-\alpha\,\beta\right)\right)\right)\right)\right)}{243\ (4+\ (\alpha-\beta)\ (\alpha+3\,\beta))^2},$$

$$-\frac{14\,\alpha}{9}-\frac{7\,\beta}{3}+\frac{4\ \left(-1+\alpha^2\right)\ (\alpha-\beta)}{4-3\,\alpha^2+2\,\alpha\,\beta+\beta^2}-$$

$$\frac{8\ (1+\alpha)\ (1+\beta)}{-4+\alpha^2+\ (-4+\beta)\ \beta-2\,\alpha\ (2+\beta)}+$$

$$\frac{8\ (-1+\alpha)\ (-1+\beta)}{-4+\alpha^2-2\,\alpha\ (-2+\beta)\ +\beta\ (4+\beta)}+$$

$$\frac{12\,\beta + 4\,\alpha\,(5+\alpha\,(-\alpha+\beta))}{9\,(4+(\alpha-\beta)\,(\alpha+3\,\beta))}, 1\}$$

(* Here $-\frac{14\,\alpha}{9}-\frac{7\,\beta}{3}+\frac{4\,(-1+\alpha^2)\,(\alpha-\beta)}{4-3\,\alpha^2+2\,\alpha\,\beta+\beta^2}-\frac{8\,(1+\alpha)\,(1+\beta)}{-4+\alpha^2+(-4+\beta)\,\beta-2\,\alpha\,(2+\beta)}+$
hier

$\frac{8\,(-1+\alpha)\,(-1+\beta)}{-4+\alpha^2-2\,\alpha\,(-2+\beta)+\beta\,(4+\beta)}+\frac{12\,\beta+4\,\alpha\,(5+\alpha\,(-\alpha+\beta))}{9\,(4+(\alpha-\beta)\,(\alpha+3\,\beta))}=-7*s$ must hold,

hence $s=\frac{1}{63}\left(14\,\alpha+21\,\beta+\frac{72\,(1+\alpha)\,(1+\beta)}{-4+\alpha^2+(-4+\beta)\,\beta-2\,\alpha\,(2+\beta)}-\frac{72\,(-1+\alpha)\,(-1+\beta)}{-4+\alpha^2-2\,\alpha\,(-2+\beta)+\beta\,(4+\beta)}+\right.$
$\left.\frac{36\,(-1+\alpha^2)\,(\alpha-\beta)}{-4+(\alpha-\beta)\,(3\,\alpha+\beta)}+\frac{4\,\alpha\,(-5+\alpha^2)-4\,(3+\alpha^2)\,\beta}{4+(\alpha-\beta)\,(\alpha+3\,\beta)}\right)$ *)

(* In this way one obtains the tentative proper monic Zolotarev
eingegebenes Objekt
polynomial of degree 7 in x, $S_{7,\alpha,\beta}[x]$, which depends solely on α and β (see the 2nd algorithm in [6], in particular Formula (12)*)

(* For Example if $s=s_0=2$ is assumed one gets
For-Schleife
from Example 2.6 in [6] the compatible values $\alpha=\alpha_0$ and $\beta=\beta_0$ *)